Indian Statistical Institute, Bangalore

B. Math (Hons.) Second Year

First Semester - Group Theory

Semester Exam Date: 08th November 2024

Maximum marks: 50 Duration: 3 hours

Section 1: Answer any four and each question carries 6 marks

1. State and Prove Cayley's Theorem.
2. If H is a proper subgroup of a finite group G, prove that $G \neq \cup_{x \in G} xHx^{-1}$.
3. Determine all groups of order pq where p and q are distinct primes.
4. Find the $o(N(H))$ where H is subgroup generated by $(1,2)(3,4)$ in S_5.
5. Determine when D_{2n} is nilpotent.
6. Show that $SL(2,\mathbb{R})$ is not solvable.

Section 2: Answer any two and each question carries 13 marks

1. (a) How many elements of order 11 and 3 are in a group of order 99?

   (b) Let H be a subgroup of a group G. Prove that $N(H)$ is the largest subgroup of G in which H is normal and prove that H is normal in G if and only if $N(H) = G$ (**Marks: 7**).
2. (a) Find the order of a permutation in S_n in terms of its cyclic decomposition.

   (b) State and prove 2nd Sylow Theorem (**Marks: 7**).
3. (a) Prove that a nontrivial normal subgroup of A_n contains a 3-cycle.

   (b) Find the exponential of the following matrices (i) $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ where $a, b \in \mathbb{R}$ (**Marks: 7**).